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HEAT CONDUCTION AND HEAT TRANSFER IN TECHNOLOGICAL PROCESSES

ESTIMATION OF CATHODE TEMPERATURE PARAMETERS BY MEANS OF A ROTATING CATHODE SPOT MODEL

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A model for calculating three-dimensional temperature fields in a cathode and estimating the parameters of the melt hollow on the surface of a hafnium cathode is proposed. A qualitative comparison of the calculation data to the photographs of the end surface of the cathode has been made.

Formulation of the Model. Modern devices for plasma cutting of metals have achieved a high level of perfection and are widely used in production processes. They provide a very high precision of cutting, and their cost is much lower than in laser devices of similar purposes. But the chief disadvantage of plasmatrons is cathode erosion, because of which one often has to replace the cathode and the nozzle of the cutter head. In this connection, investigations of the mechanism of erosion of the cathode and of ways of prolonging its service life are being carried out continually.

All erosion mechanisms are connected with the high temperature arising on the cathode surface, whose extreme values are attained in the cathode spot. Inaccuracies in estimating the temperature parameters of the latter complicate the search for the main mechanism of erosion. Experimental measurements of the cathode spot temperature present a very difficult problem, which is practically in the design of plasma cutters.

Modeling of the thermal state of the cathode is as difficult as experimental studies if one seeks to create a self-consistent model of all physical processes. Without considering the processes in the plasma, we can formulate a relatively simple model with the use of a model boundary condition simulating the interaction with the plasma. Such an approach makes it possible to carry out thermal calculations; however, the legitimacy of the used boundary condition requires validation. Since direct testing is impracticable, one can compare predictions of the model to experimental observations.

In the present paper, we propose an algorithm for calculating the temperature fields in the cathode which permits taking into account the regular motion of the cathode spot. The axisymmetric design of a cathode whose dimensions fit one of the commercial plasma devices is considered. The cathode has a refractory hafnium insert of diameter $2 \cdot 10^{-3}$ m which is pressed in the water-cooled copper body of the cathode. Figure 1 schematically represents the calculation domain with five boundaries in the cylindrical system of coordinates; the hafnium insert is marked by solid color.

The principal assumption of the model is the assumption that *heat is conveyed to the cathode through the cathode spot*; the role of the other parts of the cathode end in the heat transfer is minimum and can be considered to be adiabatic. To specify the thermal flow on the cathode spot surface, let us reason as follows. When the plasmatron is in operation, the electric power supply system works so that the direct current of the arc is kept constant, I_a = const. Since the current is largely determined by the emission of electrons, this condition can be written in the form of the integral of the thermoelectron emission current density over the emitting surface S:

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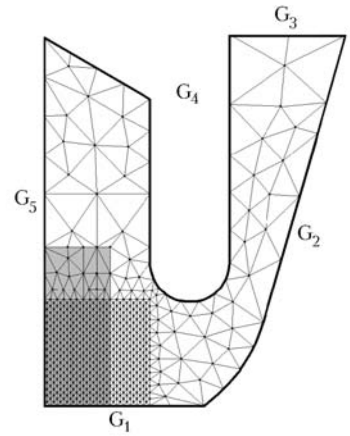

Fig. 1. Scheme of the computational domain bounded by the surfaces: G_1, arc and gas-contacting end (operating) surface; G_2, surface cooled by the input gas flow; G_3, cut-off part of the copper cathode body, considered as adiabatic because of the smallness of the heat flows; G_4, water-cooled surface; G_5, symmetry axis.

$$I_a = \int_S AT^2 \exp\left(-\frac{\Psi}{kT}\right) ds\,, \quad A = \frac{4\pi e m_e k^2}{h^3}\,. \tag{1}$$

Relation (1) has a very strong temperature dependence, i.e., only a hot cathode spot will emit electrons and be subjected to ion bombardment. This reasoning is a justification for the above assumption that heat is conveyed mainly through the cathode spot. We can further assume that the density of the heat flow to the cathode spot surface is proportional to the emission current density

$$-\lambda \nabla T \,|_S = BAT^2 \exp\left(-\frac{\Psi}{kT}\right) \tag{2}$$

or that the heat flow density in the cathode spot is constant

$$-\lambda \nabla T \,|_S = J_{\text{c.s}}\,. \tag{3}$$

In both cases, the constant B or $J_{\text{c.s}}$ should be selected either in each time step or by iteration (for a stationary formulation of the problem) so that condition (1) is fulfilled. We cannot formulate any other serious physical arguments in favor of one of these conditions. The results obtained in using these conditions are practically the same, since the total energy is controlled by condition (1). Since there were no serious reasons for choosing the condition, all calculations presented in this paper were performed with the use of the simple condition (3). The following three assumptions predetermine the motion parameters of the cathode spot which:

1) is thought to be a circle with a constant diameter (depending on the current strength);
2) rotates about the cathode axis at a constant angular velocity (depends on the gas flow swirl);
3) moves in a circle of constant radius.

These assumptions result in three parameters: $d_{\text{c.s}}$, $\omega_{\text{c.s}}$, and $R_{\text{c.s}}$, whose choice should be made on the basis of comparison between calculated and experimental data.

To describe the evolution of the temperature field, we use the nonstationary heat conduction equation without a source in the cylindrical coordinate system

$$c\rho \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\lambda r \frac{\partial T}{\partial r} + \frac{\partial}{\partial z}\lambda \frac{\partial T}{\partial z} + \frac{1}{r^2}\frac{\partial}{\partial \varphi}\lambda \frac{\partial T}{\partial \varphi}\,. \tag{4}$$

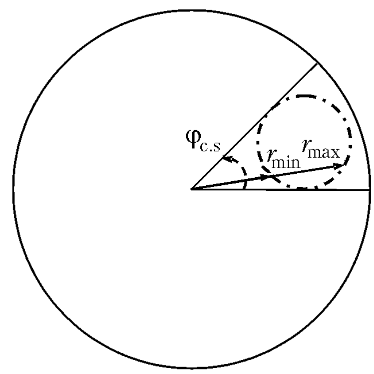

Fig. 2. Scheme of specifying the boundary condition on the surface $z = 0$ for Eq. (6).

We have neglected the heat sources connected with the Joule heat release, since it will occur in the region with the maximal current density, i.e., in the thin layer near the cathode spot. The definition of the heat flow contains a rather large number of conditionalities, and it may be presumed that it also takes into account the Joule heat since it is localized near the surface.

If we go over to a coordinate system rotating about the symmetry axis of the cathode with angular velocity $\omega_{c.s}$, i.e., $\varphi \equiv \varphi_{new} = \varphi_{old} + \omega_{c.s}t$, then the cathode spot will already be stationary, and Eq. (4) will take on the form

$$c\rho \frac{\partial T}{\partial t} + c\rho\omega_{c.s} \frac{\partial T}{\partial \varphi} = \frac{1}{r} \frac{\partial}{\partial r} \lambda r \frac{\partial T}{\partial r} + \frac{\partial}{\partial z} \lambda \frac{\partial T}{\partial z} + \frac{1}{r^2} \frac{\partial}{\partial \varphi} \lambda \frac{\partial T}{\partial \varphi} . \tag{5}$$

In such a formulation the problem remains three-dimensional and nonstationary, i.e., very unwieldy for solving. The next simplification is the search for a stationary solution. In this case, thermal conditions under which the heat entering the cathode is equal to the heat carried over by the cooling water are simulated. In the approach under consideration, we simulate not the initial heating of the cathode but its final (hottest) state. The further simplification of Eq. (5) is associated with the rejection of the heat conduction along the φ coordinate, since its contribution to the energy transfer is small (except for a certain except for certain region near the axis) compared to the convective transfer. In this approximation, Eq. (5) becomes parabolic with a marching φ coordinate

$$c\rho\omega_{c.s} \frac{\partial T}{\partial \varphi} = \frac{1}{r} \frac{\partial}{\partial r} \lambda r \frac{\partial T}{\partial r} + \frac{\partial}{\partial z} \lambda \frac{\partial T}{\partial z} . \tag{6}$$

Equation (6) is analogous to the nonstationary two-dimensional problem in which the role of the time coordinate is played by the angle φ. The formulated model requires the giving the initial temperature distribution in the plane (r, z) at the start value of the angle φ. It is impossible to find the correct initial temperature distribution, but since the temperature fields at $\varphi = 0$ and $\varphi = 2\pi$ should be equal, the initial distribution will be corrected in the course of iterations. By the iteration is meant the calculation of the temperature fields in the plane (r, z) for the range of angles φ from 0 to 2π. The convergence should be controlled by comparing the temperature fields at $\varphi = 0$ and $\varphi = 2\pi$, as well as by comparing the equality of the heat entering the cathode to the heat carried over by the cooling water. If the iterations are convergent, then they converge to the solution of Eq. (6).

Boundary conditions. Figure 2 shows the scheme of specifying the boundary condition on the surface $z = 0$ for a cathode spot of diameter $d_{c.s}$ whose center moves in a circle of radius $R_{c.s}$:

$$-\lambda \frac{\partial T}{\partial z} = \begin{cases} J_{c.s}, & 0 < \varphi < \varphi_{c.s}, \quad r_{min}(\varphi) < r < r_{max}(\varphi) ; \\ 0, & \text{in the other cases} . \end{cases} \tag{7}$$

Thus, in the range of angles $0 < \varphi < \varphi_{c.s}$ for each angle there are two radii: $r_{min}(\varphi)$ and $r_{max}(\varphi)$ defining the cathode spot boundaries.

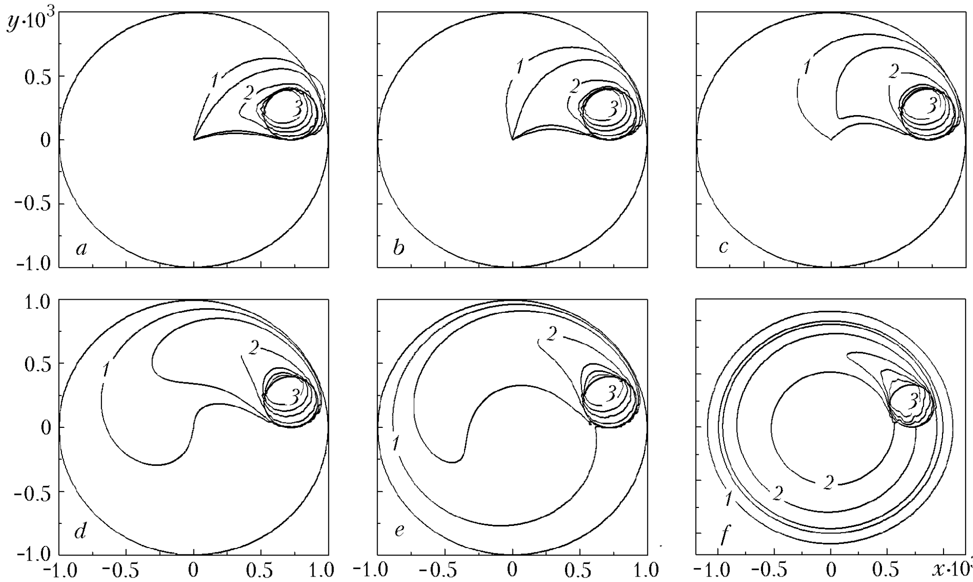

Fig. 3. Isotherms calculated on the hafnium surface for various rotational velocities of the spot [a) 5 rps; b) 10; c) 20; d) 50; e) 100; f) 1000]; 1) 1073 K; 2) 2503; 3) 4273. $D_{\mathrm{Hf}} = 2 \cdot 10^{-3}$ m; $d_{\mathrm{c.s}} = 0.4 \cdot 10^{-3}$ m; $R_{\mathrm{c.s}} = 0.75 \cdot 10^{-3}$ m; $I_{\mathrm{c.s}} \approx 200$ A.

Parametric Calculations. An important parameter of the model is the angular rotational velocity of the cathode spot $\omega_{\mathrm{c.s}}$, and it should be remembered that the applicability of the model (neglect of the heat conduction along the φ coordinate) is associated with the fulfillment of the condition $c\rho\omega_{\mathrm{c.s}}T >> \frac{\lambda}{r^2}\frac{\partial T}{\partial \varphi}$, which is more convenient to rewrite in the form $r >> \frac{\lambda}{c\rho\omega_{\mathrm{c.s}}T}\nabla_{\varphi}T$. Obviously, with increasing $\omega_{\mathrm{c.s}}$ the range of applicability of the model will widen.

The change in the form of six isotherms (1073, 1353, 2503, 3273, 3773, 4273 K) with increasing rotational velocity of the cathode spot is demonstrated by Fig. 3. (rotating coordinate system, the cathode spot is given by a circle). In varying the parameter $\omega_{\mathrm{c.s}}$, the other parameters of the problem remained constant. In Fig. 3a–e, all chosen isotherms are inside the hafnium element depicted by an outer circle. In Fig. 3f, the isotherm 1073 K appeared outside the hafnium element; therefore, the size of this illustration differs from the others.

Analyzing the isotherms in Fig. 3, it should be noted that their form with a point in the center of symmetry obtained in calculations with the cathode spot rotating with velocities 5, 10, and 20 rps is explained by the inaccuracy of the model at small distances from the axis. The high density at the "rear" boundary of the spot (clockwise movement of the spot) for $\omega_{\mathrm{c.s}}$ = 5 and 10 rps points to the expediency of taking into account the angular heat conduction in these regions. At rotations faster than 50 rps the two-dimensional computational algorithm used gives apparently a fairly adequate solution of the formulated problem. For slower rotations, the algorithm can be modified and the range of its applicability can be widened. The heat conduction along the φ coordinate can be replaced by a heat source calculated from the temperature fields of the previous iteration. Besides the iteration account of the angular heat conductivity, it is necessary to segregate the small diameter cylinder into a separate region since in it the proposed account of the angular heat conductivity is impossible in principle. This region should be regarded as isothermal in the plane (r, φ) with a temperature change only along the z coordinate. The proper choice of the diameter of this region and the correct joining, as to the heat flows, of the one-dimensional solution to the solution in the main region enables one to obtain reliable results for reasonably small values of $\omega_{\mathrm{c.s}}$. Such a modification of the algorithm should be performed only if the values of the parameter $\omega_{\mathrm{c.s}}$ for the investigated device are not high enough, say, less that 50 rps, otherwise the considered variant of the algorithm is suitable.

The form of the temperature isolines on the cathode end in Fig. 3 in no way approaches the axisymmetric form even when the cathode spot rotates at a velocity of 1000 rps. Although at such a value of $\omega_{\mathrm{c.s}}$ an annular domain of melt is observed, the electron emission is still localized in the spot and near it.

TABLE 1. Calculated Characteristics of the Cathode Spot ($d_{c.s} = 0.4 \cdot 10^{-3}$ m, $R_{c.s} = 0.75 \cdot 10^{-3}$ m, $D_{Hf} = 2 \cdot 10^{-3}$ m, $I \approx 200$ A)

ω, rps	$J_{c.s}$, $\cdot 10^{-8}$ W/m^2	T_{max}, K	$z_{max} \cdot 10^{-3}$, m	$I_{c.s}$, A	I_{total}, A	Q_{theor}, W	$Q_{calc,in}$, W
5	4.27	4999	0.174	199.8	199.8	53.66	53.58
10	4.976	5066	0.1485	199.7	200.2	62.53	62.47
20	6.03	5172	1.1246	200.6	202.5	75.78	75.75
50	8.23	5293	0.0945	200.3	206.3	103.4	103.3
100	10.625	5303	0.0755	198.0	205.9	133.5	133.2
1000	21.30	4865	0.056	200	229	268	270
1000	20.95	4808	0.055	174	200	263	267
10 000	28.53	4320	0.126	77	200	359	357
∞	2.2	3707	0.155		200	414.7	416

In performing parametric calculations with a variation of the rotation frequency, the temperature characteristics of the cathode spot have been obtained (Table 1). The emission currents have been calculated over the cathode spot surface and over the entire surface of the cathode, and the theoretical heat power has the form $Q_{theor} = J_{c.s}\pi r_{c.s}^2$. For comparison the line with $\omega = \infty$ presents the data of the axisymmetric modeling when both the heat flow and the emission current are considered to be distributed over the annular domain with $r_{min} = R_{c.s} - \frac{d_{c.s}}{2}$ and $r_{max} = R_{c.s} + \frac{d_{c.s}}{2}$.

The results of the analysis of the data of Table 1 enable us to postulate that the axisymmetric modeling gives understated values of the maximum temperature and the heat current density but an overstated value of the total heat input to the cathode. It is impossible to follow correctly the change-over from the rotating spot model to the axisymmetric model by sequentially increasing the rotational velocity of the spot because of the fundamental difference of the boundary condition of the heat input. The rotational velocity of 1000 rps in the process under consideration is the upper limit for the application of the rotating spot model. An indication of the violation of the model assumptions is a notable difference between $I_{c.s}$ and I_{total}. When the condition $I_{c.s} = 200$ A is held constant with increasing ω, then it is necessary to increase the heat flow density $I_{c.s}$, i.e., the total heat input, which leads to a heating of a larger area of the cathode surface, and the condition $I_{total} \approx I_{c.s}$ is violated. This means that in a real situation certain changes should occur in the behavior of the cathode spot, e.g., in its size, rotation frequency, and the form of its trajectory.

Replacing the condition $I_{c.s} = 200$ A by the condition $I_{total} = 200$ A partly improves the situation. The two lines for 1000 and 10,000 rps in Table 1 demonstrate the results of calculations with such a modification of the condition for the emission current. A more correct variant with the replacement of the boundary heat input condition required marked changes in the algorithm and was not realized, since in the formulation presented the model describes only the thermal pattern of the initial (very short) phase of the new cathode.

Comparison of the Calculated Data with the Experimental Information. To analyze the practicability of the proposed scheme of modeling, we took photographs with a microscope of the end surface and a metallographic axial section of new hafnium cathodes upon switching on an arc for different times. Figure 4 shows the photographs of the cathode end taken after the shortest possible for the equipment cutting time — 0.05 sec. The images are identical, but Fig. 4b contains two isotherms and the boundary of a hafnium insert having a diameter of $2 \cdot 10^{-3}$ m.

In the photographs, regions evidently subjected to melting can be seen. In these regions, we can notice bubbles formed and trajectories connected with the rotation. Outside the "smooth melt" region we can see melt ejected out of this region by the centrifugal forces and solidified in the form of irregular particles of various sizes. Near the axis a region that did not melt is observed. The form of the "smooth" melt resembles the melting isotherm at 1000 rps in Fig. 3, i.e., it may be suggested that the rotation of the cathode spot was fast enough for the annular melt region to be formed. On the other hand, if the instantaneous melt region did not close into a ring but completed several rotations, then the photograph of the solidified melt would possibly be the same.

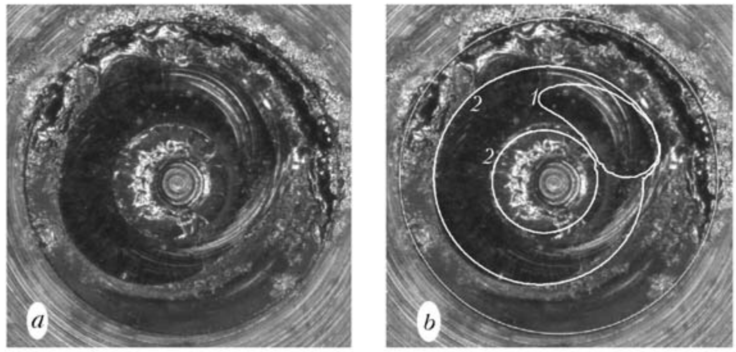

Fig. 4. Photographs of the end surface of the cathode after 0.05 sec of cutting with marked calculated boundaries of the melt and the hafnium insert.

TABLE 2. Characteristics of the Cathode Spot Calculated for Two Rotational Velocities at $I_{c.s}$ = 120 A

ω, rps	$J_{c.s}$, W/m^2	T_{max}, K	$z_{max}\cdot10^{-3}$, m	I_{total}, A	Q_{theor}, W	$Q_{calc,in}$, W
100	$8.58\cdot10^8$	5008	0.084	124	108	107
1000	$1.56\cdot10^9$	4828	0.065	136	195	194

When the cathode spot rotates at a velocity of 100 rps, then in the cutting time of 0.05 sec the spot will complete five rotations. Such a rotational velocity was taken as a "minimum estimate" for the considered experiment. For a maximum rotational velocity, we took the value of 1000 rps providing the annular form of the melting isotherm. In performing calculations, we fixed the operating current of 120 A and varied the cathode spot characteristics $d_{c.s}$ and $R_{c.s}$ in order to obtain a fairly good similarity in form with the "smooth" melt in the photograph at $\omega_{c.s}$ = 1000 rps. Figure 4b shows two melting isotherms of hafnium (2503 K) for $\omega_{c.s}$ = 100 rps (curve 1) and 1000 rps (curves 2) at equal cathode spot parameters: $d_{c.s} = 0.4\cdot10^{-3}$ m and $R_{c.s} = 0.575\cdot10^{-3}$ m. The faster-rotation variant is preferable. The parameters of the solutions of these two variants are presented in Table 2. The calculated values of T_{max} and z_{max} characterizing the melt differ, but this difference is insignificant. The isotherms superimposed on the photograph were preliminarily turned and shifted, since the real mechanical trajectory of the cathode spot is elliptical rather than circular with the center of ellipse shifted from the cathode axis.

Analysis of the picture of the solidified melt will make it possible to draw certain conclusions about the initial stage of operation of the cathode when the erosion of the operating surface has affected little its form. Comparison with the calculation data is correct only at this stage, since the model does not take into account the erosion, i.e., the cathode geometry remains unaltered. Besides the above-mentioned ellipticity of the cathode spot trajectory, in the photograph ejection of a considerable amount of melt outside the ellipse is observed, which is most probably due to the action on the liquid metal of the rotating working gas. The state of the melt and its mechanical carry-over largely determine the rate of erosion of the cathode at this stage. A change in the form of the cathode surface will lead to a change in the trajectory of the cathode spot and heat flows into the hafnium. As the elliptic hollow deepens, the near-axis unmelted region of the hafnium observed in Fig. 4 will be heated to a higher and higher temperature and finally melt. The photographs in Fig. 5 pertain to cathodes with cutting times of 0.5 and 20 sec at the same operating current of 120 A and demonstrate a situation with a hemispherical hollow.

Figure 6 shows the photographs of the axial sections of cathodes near the operating end surface. The cathode in Fig. 6a operated only for 0.05 sec and traces of erosion are hardly visible. Figure 6b shows the erosion after 20 sec of cutting where the hollow formed mainly due to the movement of the melt from the central region to the periphery is clearly seen. The ring billow formed noticeably extends above the initial surface of the cathode.

The change-over from the initial surface of the hafnium insert to a surface with a hollow is very rapid (it takes less than 1 sec). Supposedly, such a change is also followed by accelerated erosion. The geometric sizes of the hollow formed should correlate with the properties of the melt and the rotational velocity of the gaseous medium. Most probably, a "correlated" geometry of the hafnium surface will be created upon deepening of the hollow so that

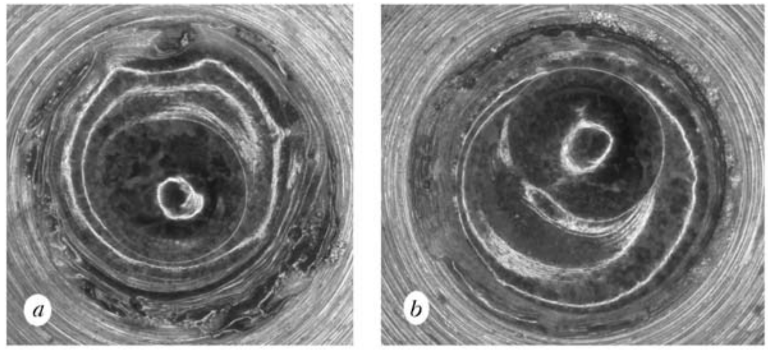

Fig. 5. Photographs of the end surfaces of the cathodes after 0.5 sec (a) and 20 sec (b) of cutting.

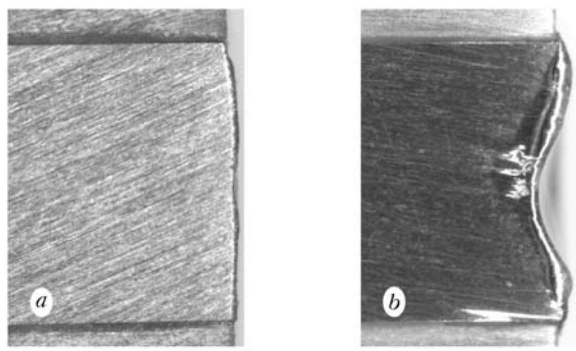

Fig. 6. Cross-sections of the cathodes after 0.05 sec (a) and 20 sec (b) of cutting.

the hafnium is completely inside the copper channel. After such a rearrangement the erosion rate should stabilize up to the moment when the next change of the mechanisms of energy transfer and, accordingly, erosion takes place. Figure 7 shows the curve of the change in the hollow depth depending on the number of equal cutting cycles, which demonstrates the establishment of a constant erosion rate after about 20 first starts. In these experiments, a constant cutting time of 26 sec in each start and equal operating current of 120 A were used.

Results and Discussion. The given photographs and the erosion curve give reason to believe that the proposed model can estimate the thermal processes only at the initial stage of operation of the cathode when its operating surface is flat. The further evolution of the temperature characteristics and form of the melt can be calculated only after creating models of erosion and motion of the melt. Proceeding from Fig. 7, it may be concluded that the complex transient processes terminate after about 15–20 starts and then the erosion rate remains constant for a long time. The description of the stage of cathode erosion with a constant rate (a few hundreds of starts) seems to be a simpler (and more important) problem than the description of the transient processes at the first starts of the device.

There exists a fairly justified opinion that at short-time starts the ablation of the hafnium mass is stronger at the moments of switching on and off of the arc and the erosion during cutting is much smaller. With such an understanding of the process, the surface pattern of the cathode can be modeled without taking into account the erosion assuming that upon each start a quasi-stationary situation is realized. A parameter of such a model will be wear of the cathode, which can be measured by the depth of the erosion hollow.

The causes and mechanisms of the formation of a hollow and its sizes and form should be described by a special model taking into account the equilibrium of the melt formed under the conditions of rotation. Such a model should give an algorithm for calculating the form of the hafnium surface (i.e., redefine the computational domain) and a formulation of the boundary condition of heat input through the cathode spot.

To create a model of melt equilibrium and calculation of the self-consistent form of the hollow, one can make use of the melt parameters calculated by the proposed method. Such parameters should be further redetermined in the course of testing and development of a new formulation of the thermal model.

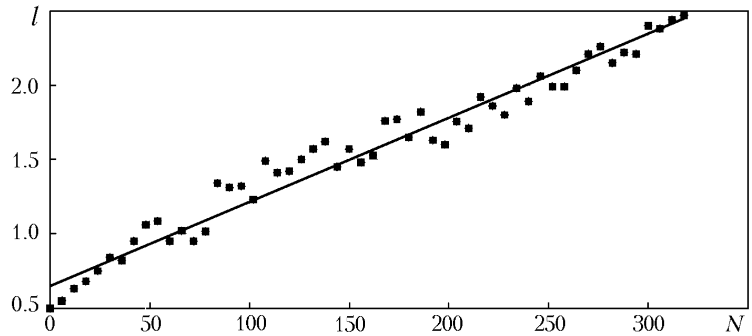

Fig. 7. Evolution of the erosion hollow depth with the number of starts. l, mm.

Conclusions. The presented algorithm of replacing the three-dimensional calculation by a sequence of two-dimensional problems has made it possible to estimate the temperature characteristics of the initial stage of operation of the cathode. Such an approach can also be used to model a quasi-stationary stage of operation of a cathode characterized by a constant erosion rate.

NOTATION

A, thermoelectron emission constant, $A/(m^2 \cdot K^2)$; B, coefficient of the linear dependence between the heat flow density and the emission current density, W/A; c, specific heat capacity, J/(kg·K); d, diameter, m; $d_{c.s}$, cathode spot diameter; D_{Hf}, hafnium insert diameter, m; ds, area element, m^2; e, electron charge, C; h, Planck constant; J/sec; I, current strength, A; J, heat flow density, W/m^2; k, Boltzmann constant, J/K; l, erosion depth, mm; m, mass, kg; N, number of starts; r, radial coordinate, m; $R_{c.s}$, radius of the circle in which the center of the cathode spot moves, m; Q, thermal energy, J; S, emitting surface, m^2; t, time, sec; T, temperature, K; x, y, coordinates, m; z, axial coordinate, m; φ, angular coordinate, rad; λ, heat conductivity coefficient, W/(m·K); ρ, density, kg/m^3; ω, angular rotational velocity, rad/sec; Ψ, work function of the electron emitted by the metal, eV/K. Subscripts: a, arc; c.s, cathode spot; calc,in, value calculated at the input; e, electron; Hf, hafnium ; min, minimum; max, maximum; new, new; old, previous; theor, theoretical value; total, total value.